

From here on out we will adopt the usual conventions used by particle physicists as opposed to relativists or string theorists or even formal field theorists..

metric convention  $\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

With that in mind we should recall the charges + his forces on us

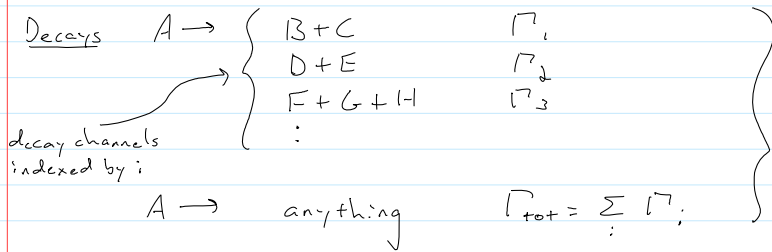
$$V^\mu = \begin{pmatrix} v^0 \\ v^1 \\ v^2 \\ v^3 \end{pmatrix} \Rightarrow V_\mu = (v^0 - v^1 - v^2 - v^3) \Rightarrow V^\mu V_\mu = v_0^2 - v_1^2 - v_2^2 - v_3^2$$

$$\mathcal{L}_{KE} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \left(\frac{m c}{\hbar}\right)^2 \phi^2 \Rightarrow \partial_\mu \partial^\mu \phi + \left(\frac{m c}{\hbar}\right)^2 \phi = 0$$

$$\mathcal{L}_{Dirac} = \hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - m c^2 \bar{\psi} \psi \Rightarrow i \gamma^\mu \partial_\mu \psi - \frac{m c}{\hbar} \psi = 0 \quad \bar{\psi} \equiv \psi^\dagger \gamma^0, \quad \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\mathcal{L}_{Proca} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \frac{1}{8\pi} \left(\frac{m c}{\hbar}\right)^2 A_\mu A^\mu \Rightarrow \partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) + \left(\frac{m c}{\hbar}\right)^2 A^\nu = 0$$

What would we like to calculate (and compare to experiment)?



$\Gamma$  are "decay rates" = probability per unit time of decaying into \_\_\_\_\_.

Now:  $dN = -\Gamma_{\text{tot}} N dt$

$N(t) = N_0 e^{-\Gamma_{\text{tot}} t}$


$\tau_{\text{avg}} = \frac{1}{\Gamma_{\text{tot}}} = \text{"lifetime"}$

set of  $\Gamma_i$

We will focus on calculating  $\{\Gamma_i\}$  from which we can get  $\Gamma_{\text{tot}}, \tau_{\text{avg}}$ .

↳ The "likelihood" of a particular set of decay products outcome:

Collisions The "likelihood" of a particular collision event  $A+B \rightarrow C+D$  is the scattering cross-section  $\sigma_i$ . The total or inclusive cross-section for  $A+B$  is  $\sigma_{\text{tot}} = \sum_i \sigma_i$ .

We can contrast with a primitive scattering like firing an arrow at a target:  In this simple case the likelihood is really determined by the actual cross-sectional area of the target.

In particle physics, scattering is much more complicated:

- soft target (interaction w/ potential)
- depends on identity of arrow
- multiple ways to successfully "hit"
- velocity dependent
- in primitive case the final state is "hit" or "no hit", whereas in particles there are many possible outcomes.

set of  $\sigma_i$

We will focus on calculating  $\{\sigma_i\}$ .

Sometimes our view is limited to a small slice of solid angle  $d\Omega$  (where detector sits), so we might instead need  $\frac{d\sigma}{d\Omega}$  which is typically only  $\theta$ -dependent.



Our interest is in relativistic, quantum mechanical calculations of  $\Gamma_i$ ;  $\sigma_i$ . This would really entail full QFT, but we will study and try to make sense of the result.

In both decays and scattering, the "likelihood" of an event is controlled by:

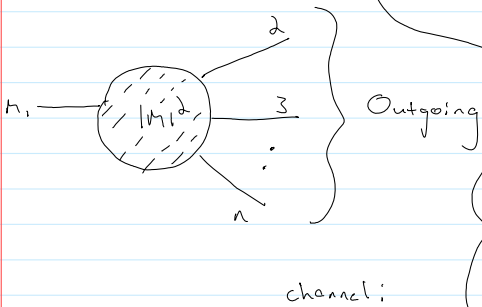
- a) Kinematics (phase-space freedom), e.g. the larger the mass difference between in and out states, the more excess kinetic energy is liberated and this can be distributed in more ways in phase-space resulting in higher likelihood.
- b) Dynamics (interactions), e.g. relative likelihoods governed by force strengths, intermediate states, etc.

These two influences actually quasi-separate in the final expressions for  $\Gamma_i$  and  $\sigma_i$ , so we can really handle them separately.

The kinematic contribution to  $\Gamma_i$ ;  $\sigma_i$  is summed up in Fermi's Golden Rule (which works for any interaction):

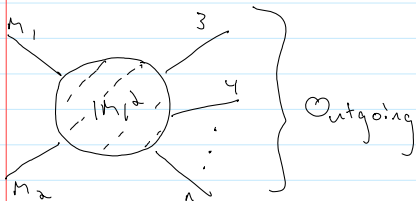
Decay:  $m_1^{\text{rest}} \rightarrow m_2 + m_3 + \dots + m_n$  (channel  $i$ ) All momenta  $p$  are 4-momenta!

$$\Gamma_i = \frac{S}{2K m_1} \int |M_i|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - \dots - p_n) \prod_{j=2}^n \frac{d^3 p_j}{(2\pi)^3} 2\pi \delta(p_j^0 - m_j c^2) \Theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$



Collisions:  $m_1 + m_2 \rightarrow m_3 + m_4 + \dots + m_n$

$$\sigma_i = \frac{S \hbar^2}{4 \sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |M_i|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - \dots - p_n) \prod_{j=3}^n \frac{d^3 p_j}{(2\pi)^3} 2\pi \delta(p_j^0 - m_j c^2) \Theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$



Note:  $S = \frac{1}{s_1!} \frac{1}{s_2!} \dots$  where  $s_i = \#$  of identical out particles of type  $i$

$|M_i|^2$  will carry all of the dynamical information.

The Golden Rule simply says that (dynamics aside) all kinematic configurations consistent with 4-momentum conservation, positive energy, and mass-shell conditions are equally likely. So the more of them there are, the higher the likelihood!!

At this point we usually can't go further since  $|M|^2$  will often depend on  $p_j$  and so we need it before integrating. But in a few special cases the kinematics is so tightly constrained that we can go a bit further.

First, we can always break up  $d^4 p_j = dp_j^0 d^3 \vec{p}_j$  and use  $\delta(p_j^2 - m_j^2 c^2) = \delta(p_j^0^2 - \vec{p}_j^2 - m_j^2 c^2)$  to perform the  $dp_j^0$  integral using the properties that:

$$\delta(x^2 - k^2) = \frac{1}{2|k|} [\delta(x-k) + \delta(x+k)] \quad k > 0$$

↑ constant

Then:

$$\Gamma_i = \frac{S}{2\pi h_i} \int |M|^2 (2\pi)^4 \delta^4(p_i - p_2 - \dots - p_n) \prod_{j=2}^n \frac{1}{2\sqrt{\vec{p}_j^2 + m_j^2 c^2}} \frac{d^3 \vec{p}_j}{(2\pi)^3}$$

$$G_i = \frac{S k^2}{4\sqrt{(p_i \cdot p_2)^2 - (m_i m_2 c)^2}} \int |M|^2 (2\pi)^4 \delta^4(p_i + p_2 - p_3 - \dots - p_n) \prod_{j=3}^n \frac{1}{2\sqrt{\vec{p}_j^2 + m_j^2 c^2}} \frac{d^3 \vec{p}_j}{(2\pi)^3}$$

Now for 2 cases that are so tightly constrained by the kinematics that we have enough  $\delta$ -functions in FGR to let us evaluate all of the integrals w/out the functional form of  $|M|^2$ .

"2-body" decay  $1 \rightarrow 2+3$

↑  
at rest!

$$\Gamma = \frac{S |\vec{p}|}{8\pi^2 h_i c} |M|^2 \text{ where } |\vec{p}| = \frac{c}{2h_i} \sqrt{h_1^4 + h_2^4 + h_3^4 - 2h_1^2 h_2^2 - 2h_1^2 h_3^2 - 2h_2^2 h_3^2}$$

↑  
magnitude of momentum of either outgoing particle  
(same since  $\vec{p}_{tot} = 0$ )

If you fix  $m_i$  and plot  $\Gamma$  as a function of  $m_2, m_3$  you will find that it grows w/ increasing mass difference.

"2-body" scattering in CM frame

$$\frac{d\sigma}{d\Omega} = \left(\frac{hc}{8\pi}\right)^2 \frac{S |M|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

↑ magnitude of momentum for either outgoing particle

↑ magnitude of momentum for either incoming particle

$1+2 \rightarrow 3+4$

$\vec{p}_{tot} = 0 = \vec{p}_{tot}$